Nome do aluno	Nº	Data
		/ / 20

Levantamento algébrico de indeterminações

1. Determine:
   - 1.1. $\lim\limits_{x\to 2}(3x^4 - x^2 + 1)$
   - 1.2. $\lim\limits_{x\to -1} \frac{2x-3}{x^2-5}$

2. Determine:
   - 2.1. $\lim\limits_{x\to +\infty} (x^3 - 3x^2)$
   - 2.2. $\lim\limits_{x\to +\infty} (1 - 3x^3 + 5x^2 - 6x)$
   - 2.3. $\lim\limits_{x\to -\infty} (6x^5 - x)$

3. Seja f uma função real de variável real do tipo $f(x) = ax^3 + 3x^2 - 5x$ com $a \in \mathbb{R}\backslash\{0\}$. Indique os valores de a para os quais:
   - 3.1. $\lim\limits_{x\to +\infty} f(x) = -\infty$
   - 3.2. $\lim\limits_{x\to -\infty} f(x) = -\infty$

4. Determine:
   - 4.1. $\lim\limits_{x\to +\infty} \sqrt{x^2+4} - \sqrt{x^2-3}$
   - 4.2. $\lim\limits_{x\to +\infty} \sqrt{x^2+6} - x$
   - 4.3. $\lim\limits_{x\to 0^+} \left(\frac{1}{x} - \frac{2}{x^2}\right)$
   - 4.4. $\lim\limits_{x\to 3} \left(\frac{x}{x-3} - \frac{3}{x^2-9}\right)$

5. Determine:
   - 5.1. $\lim\limits_{x\to +\infty} \frac{3}{2x-1}$
   - 5.2. $\lim\limits_{x\to +\infty} \frac{3x+1}{x-4}$
   - 5.3. $\lim\limits_{x\to -\infty} \frac{5x^3-2}{x^4}$
   - 5.4. $\lim\limits_{x\to -\infty} \frac{7x^4-5x+3}{3x^2+2x+1}$

6. Determine:
   - 6.1. $\lim\limits_{x\to +\infty} \frac{\sqrt{x^2+1}}{3x-1}$
   - 6.2. $\lim\limits_{x\to -\infty} \frac{\sqrt{x^2+1}}{3x-1}$
   - 6.3. $\lim\limits_{x\to +\infty} \frac{\sqrt{2x+3}-\sqrt{x-1}}{3x-2}$

7. Determine:
   - 7.1. $\lim\limits_{x\to 4} \frac{5x-20}{16-x^2}$
   - 7.2. $\lim\limits_{x\to -1} \frac{3x^2+5x+2}{x^2-1}$
   - 7.3. $\lim\limits_{x\to 1} \frac{x^3-x^2-x+1}{x^2-2x+1}$
   - 7.4. $\lim\limits_{x\to 0^-} \frac{2x^2-2x}{x^4}$

8. Calcule para que valores de k o seguinte limite é um número real:

$$\lim_{x\to -2} \frac{x^2 + kx + 2}{x^2 - 4}$$

9. Determine:

9.1. $\lim_{x\to 0}\frac{|x|}{x^2}$

9.2. $\lim_{x\to 2}\frac{|x^2-4|}{x+2}$

9.3. $\lim_{x\to 1}\frac{5-\sqrt{26-x}}{x-1}$

9.4. $\lim_{x\to 2}\frac{\sqrt{x+2}-\sqrt{6-x}}{x-2}$

10. Determine, caso exista:

10.1. $\lim_{x\to -5}\left(\frac{1}{x+5}\times(x^2-25)\right)$

10.2. $\lim_{x\to 0}(|x|x^{-1})$

11. Considere a função real de variável real de domínio $\mathbb{R}^+$ definida por:

$$f(x)=\begin{cases}\dfrac{x-1}{x-\sqrt{x}} & se\ 0<x<1\\ 2 & se\ x=1\\ \dfrac{-6x+6x^3}{x^2+4x-5} & se\ x>1\end{cases}$$

Determine, caso exista:

11.1. $\lim_{x\to 0^+} f(x)$

11.2. $\lim_{x\to 1} f(x)$

11.3. $\lim_{x\to +\infty} f(x)$

Soluções

1.

1.1.

$$\lim_{x \to 2}(3x^4 - x^2 + 1) = 3 \times 2^4 - 2^2 + 1 = 48 - 4 + 1 = 45$$

1.2.

$$\lim_{x \to -1} \frac{2x - 3}{x^2 - 5} = \frac{2 \times (-1) - 3}{(-1)^2 - 5} = \frac{-5}{-4} = \frac{5}{4}$$

2.

2.1.

$$\lim_{x \to +\infty} (x^3 - 3x^2) = \lim_{x \to +\infty} \left[x^3\left(1 - \frac{3}{x}\right)\right] = +\infty \times 1 = +\infty$$

2.2.

$$\lim_{x \to +\infty} (1 - 3x^3 + 5x^2 - 6x) = \lim_{x \to +\infty} \left[x^3\left(\frac{1}{x^3} - 3 + \frac{5}{x} - \frac{6}{x^2}\right)\right] =$$
$$= +\infty \times (-3) = -\infty$$

2.3.

$$\lim_{x \to -\infty} (6x^5 - x) = \lim_{x \to -\infty} \left[x^5\left(6 - \frac{1}{x^4}\right)\right] = -\infty \times 6 = -\infty$$

3.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} (ax^3) = \pm\infty \times a$$

3.1. $a \in \left]-\infty, 0\right[$

3.2. $a \in \left]0, +\infty\right[$

4.

4.1.

$$\lim_{x \to +\infty} \left(\sqrt{x^2+4} - \sqrt{x^2-3}\right) =$$
$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2 + 4} - \sqrt{x^2 - 3}\right)\left(\sqrt{x^2 + 4} + \sqrt{x^2 - 3}\right)}{\left(\sqrt{x^2 + 4} + \sqrt{x^2 - 3}\right)} =$$
$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x^2 + 4}\right)^2 - \left(\sqrt{x^2 - 3}\right)^2}{\sqrt{x^2 + 4} + \sqrt{x^2 - 3}} \underset{(1)}{=} \lim_{x \to +\infty} \frac{x^2 + 4 - x^2 + 3}{\sqrt{x^2 + 4} + \sqrt{x^2 - 3}} =$$
$$= \frac{7}{+\infty} = 0$$

(1) Para $x > \sqrt{3}$, $\left(\sqrt{x^2 + 4}\right)^2 = x^2 + 4$ e $\left(\sqrt{x^2 - 3}\right)^2 = x^2 - 3$.

4.2.

$$\lim_{x \to +\infty} \left(\sqrt{x^2 + 6} - x\right) = \lim_{x \to +\infty} \frac{\left(\sqrt{x^2 + 6} - x\right)\left(\sqrt{x^2 + 6} + x\right)}{\sqrt{x^2 + 6} + x} =$$
$$= \lim_{x \to +\infty} \frac{x^2 + 6 - x^2}{\sqrt{x^2 + 6} + x} = \frac{6}{+\infty} = 0$$

4.3.

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{2}{x^2}\right) = \lim_{x \to 0^+} \frac{x - 2}{x^2} = \frac{-2}{0^+} = -\infty$$

4.4.

$$\lim_{x \to 3^-}\left(\frac{x}{x-3} - \frac{3}{x^2-9}\right) = \lim_{x \to 3^-}\left(\frac{x(x+3)-3}{x^2-9}\right) = \lim_{x \to 3^-} \frac{x^2+3x-3}{x^2-9} = \frac{15}{0^-} = -\infty$$

5.

5.1.

$$\lim_{x \to +\infty} \frac{3}{2x-1} = \frac{3}{+\infty} = 0$$

5.2.

$$\lim_{x \to +\infty} \frac{3x+1}{x-4} = \lim_{x \to +\infty} \frac{x\left(3+\frac{1}{x}\right)}{x\left(1-\frac{4}{x}\right)} = \lim_{x \to +\infty} \frac{3+\frac{1}{x}}{1-\frac{4}{x}} = 3$$

5.3.

$$\lim_{x \to -\infty} \frac{5x^3-2}{x^4} = \lim_{x \to -\infty} \frac{5x^3\left(1-\frac{2}{5x^3}\right)}{x^4} = \lim_{x \to -\infty} \frac{5\left(1-\frac{2}{5x^3}\right)}{x} = \frac{5(1-0)}{-\infty} = 0$$

5.4.

$$\lim_{x \to -\infty} \frac{7x^4-5x+3}{3x^2+2x+1} = \lim_{x \to -\infty} \frac{7}{3}x^2 = \frac{7}{3} \times (+\infty) = +\infty$$

6.

6.1.

$$\lim_{x \to +\infty} \frac{\sqrt{x^2+1}}{3x-1} = \lim_{x \to +\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{3x-1} = \lim_{x \to +\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{3x-1} =$$

$$= \lim_{x \to +\infty} \frac{x\sqrt{1+\frac{1}{x^2}}}{x\left(3-\frac{1}{x}\right)} = \frac{\sqrt{1}}{3} = \frac{1}{3}$$

6.2.

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{3x-1} = \lim_{x \to -\infty} \frac{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{3x-1} = \lim_{x \to -\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{3x-1} =$$

$$= \lim_{x \to -\infty} \frac{-x\sqrt{1+\frac{1}{x^2}}}{x\left(3-\frac{1}{x}\right)} = \frac{-\sqrt{1}}{3} = -\frac{1}{3}$$

6.3.

$$\lim_{x \to +\infty} \frac{\sqrt{2x+3}-\sqrt{x-1}}{3x+2} =$$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{2x+3}-\sqrt{x-1}\right)\left(\sqrt{2x+3}+\sqrt{x-1}\right)}{(3x+2)\left(\sqrt{2x+3}+\sqrt{x-1}\right)} =$$

$$= \lim_{x \to +\infty} \frac{2x+3-x+1}{(3x+2)\left(\sqrt{2x+3}+\sqrt{x-1}\right)} =$$

$$= \lim_{x \to +\infty} \frac{x+4}{3x+2} \times \lim_{x \to +\infty} \frac{1}{\left(\sqrt{2x+3}+\sqrt{x-1}\right)} = \frac{1}{3} \times 0 = 0$$

7.

7.1.

$$\lim_{x \to 4} \frac{5x - 20}{16 - x^2} = \lim_{x \to 4} \frac{5(x-4)}{(4-x)(4+x)} = \lim_{x \to 4} \frac{-5}{4+x} = -\frac{5}{8}$$

7.2.

$$\lim_{x \to -1} \frac{3x^2 + 5x + 2}{x^2 - 1} \underset{(1)}{=} \lim_{x \to -1} \frac{(x+1)(3x+2)}{(x-1)(x+1)} = \lim_{x \to -1} \frac{3x+2}{x-1} = \frac{-1}{-2} = \frac{1}{2}$$

(1) Cálculos auxiliares:

	3	5	2
−3		−3	−2
	3	2	0

$3x^2 + 5x + 2 = (x+1)(3x+2)$

7.3.

No cálculo de $\lim_{x \to 1} \frac{x^3 - x^2 - x + 1}{x^2 - 2x + 1}$ obtém-se a indeterminação $\frac{0}{0}$.

Como o numerador e o denominador são divisíveis por $x - 1$, aplicando a regra de Ruffini:

	1	−1	−1	1
1		1	0	−1
	1	0	−1	0

	1	−2	1
1		1	−1
	1	−1	0

Logo:

$$\lim_{x \to 1} \frac{x^3 - x^2 - x + 1}{x^2 - 2x + 1} = \lim_{x \to 1} \frac{(x-1)(x^2-1)}{(x-1)(x-1)} =$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1}(x+1) = 2$$

7.4.

$$\lim_{x \to 0^-} \frac{2x^2 - 2x}{x^4} = \lim_{x \to 0^-} \frac{x(2x-2)}{x^4} = \lim_{x \to 0^-} \frac{2x-2}{x^3} = \frac{-2}{0^-} = +\infty$$

8.

Como -2 é um zero do denominador, pretende-se que também o numerador se anule em -2. Então:

$$(-2)^2 - 2k + 2 = 0 \Leftrightarrow k = 3$$

Logo:

$$\lim_{x \to -2} \frac{x^2 + 3x + 2}{(x-2)(x+2)} = \lim_{x \to -2} \frac{(x+2)(x+1)}{(x-2)(x+2)} =$$

$$= \lim_{x \to -2} \frac{x+1}{x-2} = \frac{-1}{-4} = \frac{1}{4} \in \mathbb{R}$$

9.

9.1.

$$\lim_{x \to 0^-} \frac{|x|}{x^2} = \lim_{x \to 0^-} \frac{-x}{x^2} = \lim_{x \to 0^-} \frac{-1}{x} = \frac{-1}{0^-} = +\infty$$

$$\lim_{x \to 0^+} \frac{|x|}{x^2} = \lim_{x \to 0^+} \frac{x}{x^2} = \lim_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = +\infty$$

Portanto, $\lim_{x \to 0} \frac{|x|}{x^2} = +\infty$.

Em alternativa:

$$\lim_{x \to 0} \frac{|x|}{x^2} = \lim_{x \to 0} \frac{|x|}{|x|^2} = \lim_{x \to 0} \frac{1}{|x|} = +\infty$$

9.2.

$$\lim_{x \to 2^-} \frac{|x^2 - 4|}{x + 2} = \frac{0}{4} = 0$$

9.3.

$$\lim_{x \to 1} \frac{5 - \sqrt{26 - x}}{x - 1} = \lim_{x \to 1} \frac{\left(5 - \sqrt{26 - x}\right)\left(5 + \sqrt{26 - x}\right)}{(x - 1)\left(5 + \sqrt{26 - x}\right)} =$$

$$= \lim_{x \to 1} \frac{x - 1}{(x - 1)\left(5 + \sqrt{26 - x}\right)} = \lim_{x \to 1} \frac{1}{5 + \sqrt{26 - x}} =$$

$$= \frac{1}{5 + \sqrt{25}} = \frac{1}{10}$$

9.4.

$$\lim_{x \to 2} \frac{\sqrt{x + 2} - \sqrt{6 - x}}{x - 2} =$$

$$= \lim_{x \to 2} \frac{\left(\sqrt{x + 2} - \sqrt{6 - x}\right)\left(\sqrt{x + 2} + \sqrt{6 - x}\right)}{(x - 2)\left(\sqrt{x + 2} + \sqrt{6 - x}\right)} =$$

$$= \lim_{x \to 2} \frac{2x - 4}{(x - 2)\left(\sqrt{x + 2} + \sqrt{6 - x}\right)} = \lim_{x \to 2} \frac{2(x - 2)}{(x - 2)\left(\sqrt{x + 2} + \sqrt{6 - x}\right)} =$$

$$= \lim_{x \to 2} \frac{2}{\sqrt{x + 2} + \sqrt{6 - x}} = \frac{2}{2\sqrt{4}} = \frac{1}{2}$$

10.

10.1.

$$\lim_{x \to -5} \left(\frac{1}{x + 5} \times (x^2 - 25) \right) = \lim_{x \to -5} \frac{(x - 5)(x + 5)}{x + 5} = \lim_{x \to -5} (x - 5) = -10$$

10.2.

$$\lim_{x \to 0^-} (|x| x^{-1}) = \lim_{x \to 0^-} \left((-x) \times \frac{1}{x} \right) = -1$$

$$\lim_{x \to 0^+} (|x| x^{-1}) = \lim_{x \to 0^+} \left(x \times \frac{1}{x} \right) = 1$$

Como os limites laterais são diferentes, não existe $\lim_{x \to 0} (|x| x^{-1})$.

11.

11.1.

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x - 1}{x - \sqrt{x}} = \frac{-1}{0^-} = +\infty$$

11.2.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{x - 1}{x - \sqrt{x}} = \lim_{x \to 1^-} \frac{(x-1)(x+\sqrt{x})}{(x-\sqrt{x})(x+\sqrt{x})} =$$

$$= \lim_{x \to 1^-} \frac{(x-1)(x+\sqrt{x})}{x(x-1)} = \lim_{x \to 1^-} \frac{x+\sqrt{x}}{x} = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{6x - 6x^3}{-x^2 - 4x + 5} = \lim_{x \to 1^+} \frac{6x(1-x^2)}{-x^2 - 4x + 5} =$$

$$= \lim_{x \to 1^+} \frac{6x(1-x)(1+x)}{-x^2 - 4x + 5}$$

Aplicando a regra de Ruffini:

	−1	−4	5
1		−1	−5
	−1	−5	0

$$-x^2 - 4x + 5 = (x - 1)(-x - 5)$$

Logo:

$$\lim_{x \to 1^+} \frac{6x(1-x)(1+x)}{-x^2 - 4x + 5} = \lim_{x \to 1^+} \frac{-6x(x-1)(x+1)}{(x-1)(-x-5)} =$$

$$= \lim_{x \to 1^+} \frac{-6x(x+1)}{-x-5} = \frac{-6 \times 2}{-1-5} = 2$$

Como $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1) = 2$, $\lim_{x \to 1} f(x) = 2$.

11.3.

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{6x - 6x^3}{-x^2 - 4x + 5} = \lim_{x \to +\infty} \frac{6x^3\left(\frac{1}{x^2} - 1\right)}{-x^2\left(1 + \frac{4}{x} - \frac{5}{x^2}\right)} =$$

$$= \lim_{x \to +\infty} \frac{6x\left(\frac{1}{x^2} - 1\right)}{-\left(1 + \frac{4}{x} - \frac{5}{x^2}\right)} = \frac{6 \times (+\infty) \times (-1)}{-1} = 6 \times (+\infty) = +\infty$$