Nome do aluno	Nº	Data
		/ / 20

Álgebra de limites de uma função

1. Determine:

1.1. $\lim_{x\to 2}(2x^4 - 3x + 10)$

1.2. $\lim_{x\to -1}\frac{4x^3-3x}{-x^4+3}$

1.3. $\lim_{x\to 0}\frac{\sqrt{x^2+1}}{(x+1)^2}$

1.4. $\lim_{x\to 0}\frac{3-4x}{x^2}$

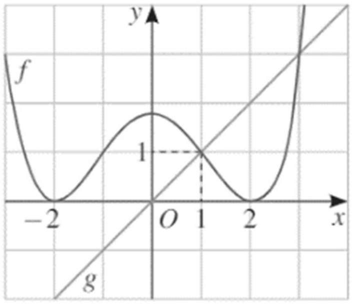
1.5. $\lim_{x\to 2}\frac{x+1}{x-2}$

1.6. $\lim_{x\to 3^+}\frac{4}{3-x}$

1.7. $\lim_{x\to -3}\frac{-2x}{x^2-9}$

1.8. $\lim_{x\to -\infty}\frac{-1}{x}$

2. Na figura ao lado estão representadas partes dos gráficos de duas funções polinomiais, f e g, de graus 4 e 1, respetivamente. A função f tem dois zeros, -2 e 2, e g tem um único zero, 0.



Sabe-se ainda que: $f(1) = g(1) = 1$.

Determine, caso existam, os limites seguintes:

2.1. $\lim_{x\to 1}\frac{f(x)}{g(x)}$

2.2. $\lim_{x\to 0}\frac{g(x)}{f(x)}$

2.3. $\lim_{x\to 2}\frac{g(x)}{f(x)}$

2.4. $\lim_{x\to +\infty}(f(x) + g(x))$

2.5. $\lim_{x\to -\infty}(f(x) \times g(x))$

2.6. $\lim_{x\to 0}\frac{f(x)}{g(x)}$

3. Considere as funções f e g, reais de variável real, definidas, respetivamente, por:

$$f(x) = x^2 - x \qquad e \qquad g(x) = \begin{cases} x+1 & se\ x \geq 1 \\ 1+2x & se\ x < 1 \end{cases}$$

3.1. Calcule $\lim_{x\to 1} f(x)$.

3.2. Calcule $\lim_{x\to 1^-} g(x)$ e $\lim_{x\to 1^+} g(x)$.

3.3. Conclua que não existe $\lim_{x\to 1} g(x)$.

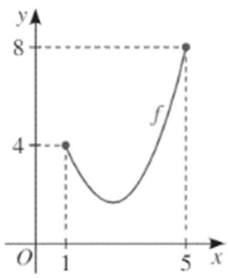
3.4. Justifique que $\lim_{x\to 1}[f(x) \times g(x)] = 0$.

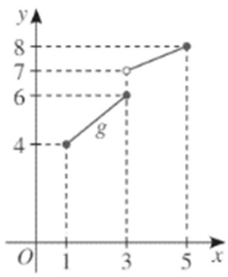
4. Considere as funções f e g de domínio $[1, 5]$, representadas graficamente nas figuras seguintes, e a função h de domínio $\mathbb{R}$ definida por $h(x) = x^2 - 9$.

Determine, justificando:

4.1. $\lim_{x\to 3}[f(x) \times h(x)]$

4.2. $\lim_{x\to 3}[g(x) \times h(x)]$





Soluções

1.

1.1.

$$\lim_{x \to 2}(2x^4 - 3x + 10) = \lim_{x \to 2}(2x^4) + \lim_{x \to 2}(-3x) + \lim_{x \to 2} 10 =$$
$$= 2\left(\lim_{x \to 2} x\right)^4 - 3\lim_{x \to 2} x + 10 = 2^5 - 3 \times 2 + 10 = 36$$

1.2.

$$\lim_{x \to -1} \frac{4x^3 - 3x}{-x^4 + 3} = \frac{\lim\limits_{x \to -1}(4x^3 - 3x)}{\lim\limits_{x \to -1}(-x^4 + 3)} = \frac{4\left(\lim\limits_{x \to -1} x\right)^3 - 3\lim\limits_{x \to -1} x}{-\left(\lim\limits_{x \to -1} x\right)^4 + \lim\limits_{x \to -1} 3} =$$
$$= \frac{4 \times (-1) - 3 \times (-1)}{-(1) + 3} = -\frac{1}{2}$$

1.3.

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 1}}{(x + 1)^2} = \lim_{x \to 0} \frac{(x^2 + 1)^{\frac{1}{2}}}{(x + 1)^2} = \frac{\lim\limits_{x \to 0}(x^2 + 1)^{\frac{1}{2}}}{\lim\limits_{x \to 0}(x + 1)^2} =$$
$$= \frac{\left[\left(\lim\limits_{x \to 0} x\right)^2 + \lim\limits_{x \to 0} 1\right]^{\frac{1}{2}}}{\left(\lim\limits_{x \to 0} x + \lim\limits_{x \to 0} 1\right)^2} = \frac{(0 + 1)^{\frac{1}{2}}}{(0 + 1)^2} = 1^{-\frac{3}{2}} = 1$$

1.4.

$$\lim_{x \to 0} \frac{3 - 4x}{x^2} = \lim_{x \to 0}\left(\frac{1}{x^2}\right) \times \lim_{x \to 0}(3 - 4x) = +\infty \times 3 = +\infty$$

1.5.

$$\lim_{x \to 2^-} \frac{x + 1}{x - 2} = \frac{\lim\limits_{x \to 2^-} x + \lim\limits_{x \to 2^-} 1}{\lim\limits_{x \to 2^-} x - \lim\limits_{x \to 2^-} 2} = \frac{3}{0^-} = -\infty$$

1.6.

$$\lim_{x \to 3^+} \frac{4}{3 - x} = \frac{\lim\limits_{x \to 3^+} 4}{3 - \lim\limits_{x \to 3^+} x} = \frac{4}{0^-} = -\infty$$

1.7.

$$\lim_{x \to -3^-} \frac{-2x}{x^2 - 9} = \frac{-2\lim\limits_{x \to -3^-} x}{\left(\lim\limits_{x \to -3^-} x\right)^2 - \lim\limits_{x \to -3^-} 9} = \frac{6}{0^+} = +\infty$$

1.8.

$$\lim_{x \to -\infty} \frac{-1}{x} = \frac{-1}{\lim\limits_{x \to -\infty} x} = \frac{-1}{-\infty} = 0$$

2.

2.1.

$$\lim_{x \to 1} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to 1} f(x)}{\lim\limits_{x \to 1} g(x)} = \frac{1}{1} = 1$$

2.2.

$$\lim_{x \to 0} \frac{g(x)}{f(x)} = \frac{\lim\limits_{x \to 0} g(x)}{\lim\limits_{x \to 0} f(x)} = \frac{0}{a} = 0 \text{, com } a \in \mathbb{R}^+$$

2.3.

$$\lim_{x \to 2} \frac{g(x)}{f(x)} = \frac{\lim\limits_{x \to 2} g(x)}{\lim\limits_{x \to 2} f(x)} = \frac{2}{0^+} = +\infty$$

2.4.

$$\lim_{x \to +\infty} \left(f(x) + g(x) \right) = \lim_{x \to +\infty} f(x) + \lim_{x \to +\infty} g(x) = +\infty + (+\infty) = +\infty$$

2.5.

$$\lim_{x \to -\infty} \left(f(x) \times g(x) \right) = \lim_{x \to -\infty} f(x) \times \lim_{x \to -\infty} g(x) = +\infty \times (-\infty) = -\infty$$

2.6.

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to 0} f(x)}{\lim\limits_{x \to 0} g(x)} = \frac{a}{0}, \text{ com } a \in \mathbb{R}^+$$

Como $\lim\limits_{x \to 0^+} \frac{f(x)}{g(x)} = +\infty \neq -\infty = \lim\limits_{x \to 0^-} \frac{f(x)}{g(x)}$, não existe $\lim\limits_{x \to 0} \frac{f(x)}{g(x)}$.

3.

3.1. $\lim\limits_{x \to 1^-} f(x) = 0$

3.2. $\lim\limits_{x \to 1^-} g(x) = 3$ e $\lim\limits_{x \to 1^+} g(x) = 2$

3.3. Não existe $\lim\limits_{x \to 1} g(x)$ porque os limites laterais são diferentes.

3.4.

$\lim\limits_{x \to 1^-} [f(x) \times g(x)] = 0 \times 3 = 0$

$\lim\limits_{x \to 1^+} [f(x) \times g(x)] = 0 \times 2 = 0$

Como $f(1) \times g(1) = 0$ e os limites laterais são iguais a 0,
$\lim\limits_{x \to 1} [f(x) \times g(x)] = 0$.

4.

4.1.

$\lim\limits_{x \to 3} [f(x) \times h(x)] = a \times 0 = 0$, com $a \in \mathbb{R}^+$

$f(3) \times h(3) = a \times 0 = 0$

Então, $\lim\limits_{x \to 3} [f(x) \times h(x)] = 0$.

4.2.

$\lim\limits_{x \to 3^-} [g(x) \times h(x)] = 6 \times 0 = 0$

$\lim\limits_{x \to 3^+} [g(x) \times h(x)] = 7 \times 0 = 0$

$g(3) \times h(3) = 6 \times 0 = 0$

Então, $\lim\limits_{x \to 3} [g(x) \times h(x)] = 0$.