Nome do aluno | Nº | Data / / 20

Primitivas – exercícios globais

1. Calcule, em intervalos convenientes, as seguintes primitivas:

1.1. $\int (6x^2 - 4x + 3)\, dx$

1.2. $\int (\cos x + x)\, dx$

1.3. $\int \left(3x - \frac{2}{x} - 3\cos x\right) dx$

1.4. $\int 2e^{3x}\, dx$

1.5. $\int \frac{6}{\sqrt{x}} dx$

1.6. $\int x^3(x^4+1)^8\, dx$

1.7. $(2^x + e^x \sin(e^x))dx$

1.8. $\int \sin x \cos^2 x\, dx$

1.9. $\int \sin x \cos x\, e^{\cos(2x)}\, dx$

2. Para as seguintes funções, calcule em $\mathbb{R}$ as respetivas primitivas cujo gráfico passa na origem do referencial.

2.1. $f(x) = x^2 e^{x^3} + 3\sin x$

2.2. $g(x) = 4\sin(2x)$

2.3. $h(x) = 5\cos^2 x - 5\sin^2 x$

3. seja f uma função diferenciável de domínio I.

3.1. Justifique que $\int 2f(x)f'(x)\, dx = f^2(x) + c, c \in \mathbb{R}$.

3.2. Utilize o resultado da alínea anterior para calcular as seguintes primitivas:

3.2.1. $\int 2(x^2-1)2x\, dx$

3.2.2. $\int \frac{\ln}{x} dx$

4. Considere a função f diferenciável de domínio I.

4.1. Mostre que $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c, c \in \mathbb{R}$.

4.2. Utilize o resultado da alínea anterior para calcular as seguintes primitivas:

4.2.1. $\int \frac{2x+3}{x^2+3x} dx$

4.2.2. $\int \frac{\sin x}{\cos x} dx$

5. Determine, em $\mathbb{R}$, $\int \cos^2 x\, dx$ e $\int \cos^3 x\, dx$.

Sugestão: comece por provar que:

- $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$
- $\cos^3 x = \cos x - \cos x \sin^2 x$

Soluções

1.

1.1.

$$\int (6x^2 - 4x + 3)\,dx = 6\int x^2\,dx + 4\int x\,dx + \int 3\,dx =$$
$$= 6\frac{x^3}{3} - 4\frac{x^2}{2} + 3x + c = 2x^3 - 2x^2 + 3x + c$$

1.2.

$$\int (\cos x + x)\,dx = \int \cos x\,dx + \int x\,dx = \sin x + \frac{x^2}{2} + c$$

1.3.

$$\int\left(3x - \frac{2}{x} - 3\cos x\right)dx = 3\int x\,dx - 2\int \frac{1}{x}\,dx - 3\int \cos x\,dx =$$
$$= \frac{3x^2}{2} - 2\ln x - 3\sin x + c\,,\ \text{para}\ x \in \left]0, +\infty\right[$$

1.4.

$$\int 2e^{3x}\,dx = 2\int e^{3x}\,dx = \frac{2}{3}\int 3e^{3x}\,dx = \frac{2}{3}\,e^{3x} + c$$

1.5.

$$\int \frac{6}{\sqrt{x}}\,dx = 6\int x^{-\frac{1}{2}}\,dx = 6 \times 2\int \frac{1}{2}x^{-\frac{1}{2}}\,dx = 12x^{-\frac{1}{2}} + c = 12\sqrt{x} + c\,,\ \text{para}\ x \in \left]0, +\infty\right[$$

1.6.

$$\int x^3(x^4+1)^8\,dx = \frac{1}{4}\int 4x^3(x^4+1)^8\,dx = \frac{1}{4}\frac{\left(x^4+1\right)^9}{9} + c = \frac{\left(x^4+1\right)^9}{36} + c$$

1.7.

$$\int \left(2^x + e^x\sin(e^x)\right)dx = \int 2^x\,dx + \int e^x\sin(e^x)\,dx = \frac{2^x}{\ln 2} - \cos(e^x) + c$$

1.8.

$$\int \sin x\cos^2 x\,dx = -\int (\cos x)'\,\cos^2 x\,dx = -\frac{\cos^3 x}{3} + c$$

1.9.

$$\int \sin x\cos x\,e^{\cos(2x)}\,dx = \frac{1}{4}\int 4\sin x\cos x\,e^{\cos(2x)}\,dx =$$
$$= -\frac{1}{4}\int -2\sin(2x)\,e^{\cos(2x)}\,dx = -\frac{1}{4}\int \left(\cos(2x)\right)'e^{\cos(2x)}\,dx =$$
$$= -\frac{1}{4}\,e^{\cos(2x)} + c$$

2.

2.1.

$$\int (x^2e^{x^3} + 3\sin x)\,dx = \int x^2e^{x^3}\,dx + 3\int \sin x\,dx =$$
$$= \frac{1}{3}\int 3x^2e^{x^3}\,dx + 3\int \sin x\,dx = \frac{1}{3}e^{x^3} - 3\cos x + c$$
$$F(0) = 0 \Leftrightarrow \frac{e^0}{3} - 3\cos 0 + c = 0 \Leftrightarrow \frac{1}{3} - 3 + c = 0 \Leftrightarrow c = \frac{8}{3}$$
$$F(x) = \frac{1}{3}e^{x^3} - 3\cos x + \frac{8}{3}$$

2.2.

$$\int 4\sin(2x)\,dx = 2\int 2\sin(2x)\,dx = -2\cos(2x) + c$$
$$G(0) = 0 \Leftrightarrow -2\cos 0 + c = 0 \Leftrightarrow -2 + c = 0 \Leftrightarrow c = 2$$
$$G(x) = -2\cos(2x) + 2$$

2.3.

$$\int (5\cos^2 x - 5\sin^2 x)\,dx = 5\int (\cos^2 x - \sin^2 x)\,dx = 5\int \cos(2x)\,dx =$$
$$= \frac{5}{2}\int 2\cos(2x)\,dx = \frac{5}{2}\sin(2x) + c$$
$$H(0) = 0 \Leftrightarrow \frac{5}{2}\sin 0 + c = 0 \Leftrightarrow c = 0$$
$$H(x) = \frac{5}{2}\sin(2x)$$

3.

3.1.

$$(f^2(x))' = 2 \times f(x)f'(x)\text{, logo, } \int 2f(x)f'(x)dx = f^2(x) + c\text{, } c \in \mathbb{R}\text{.}$$

3.2.

3.2.1.

$$\int 2(x^2 - 1)2x\,dx = 2\int (x^2 - 1)(x^2 - 1)'\,dx = (x^2 - 1)^2 + c$$

3.2.2.

$$\int \frac{\ln x}{x}dx = \frac{1}{2}\int\left(2\ln x \times \frac{1}{x}\right)dx = \frac{(\ln x)^2}{2} + c\text{, para } x > 0$$

4.

4.1.

$$(\ln|f(x)|)' = \frac{f'(x)}{f(x)}\text{, logo, } \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c\text{, } c \in \mathbb{R}$$

4.2.

4.2.1.

$$\int \frac{2x + 3}{x^2 + 3x}dx = \ln|x^2 + 3x| + c$$

4.2.2.

$$\int \frac{\sin x}{\cos x}dx = -\ln|\cos x| + c$$

5.

$$\cos(2x) = \cos^2 x - \sin^2 x \Leftrightarrow \cos(2x) = \cos^2 x - 1 + \cos^2 x \Leftrightarrow$$
$$\Leftrightarrow \cos(2x) = 2\cos^2 x - 1 \Leftrightarrow \cos(2x) = \frac{\cos(2x) + 1}{2}$$

$$\int \cos^2 x\,dx = \int \frac{\cos(2x) + 1}{2}dx = \frac{1}{2}\int (\cos(2x) + 1)\,dx =$$
$$= \int \frac{1}{2}dx + \frac{1}{2}\int \cos(2x)\,dx = \int \frac{1}{2}dx + \frac{1}{4}\int 2\cos(2x)\,dx =$$
$$= \frac{x}{2} + \frac{1}{4}\sin(2x) + c$$

$$\cos^3 x = \cos x \times \cos^2 x = \cos x \times (1 - \sin^2 x) = \cos x - \cos x \sin^2 x$$

$$\int \cos^3 x dx = \int (\cos x - \cos x \sin^2 x) dx = \int \cos x dx - \int \cos x \sin^2 x dx =$$

$$= \sin x - \sin^3 \frac{x}{3} + c$$