Nome do aluno	Nº	Data / / 20

Função logaritmo de base positiva diferente de 1

1. Determine o valor de:

1.1. $\log_2 32$

1.2. $\log_{\frac{1}{3}} 3$

1.3. $\log_{\sqrt{5}} 5$

1.4. $\log_\pi \left(\frac{1}{\pi^2}\right)$

1.5. $\log_4 2$

2. Determine o valor exato de:

2.1. $5^{3\times\log_5 2}$

2.2. $\pi^{\log_{32} 1}$

2.3. $\log_e(\log_{10} 10)$

2.4. $\sqrt{2}^{4+\log_2 5}$

2.5. $\log_{\frac{1}{3}}(3\times\sqrt{27})$

2.6. $\log_{125}\sqrt{5}$

3. Resolva, em $\mathbb{R}$, as equações:

3.1. $\log_2 x = -2$

3.2. $2 = \log_x 3$

3.3. $\log_{x-1} 4 = 1$

3.4. $\log_x(2x-1) = 2$

3.5. $x^3 = -4$

3.6. $3^x = 4$

3.7. $e^{2x} = 2$

3.8. $e^{x-1}\times 2^x - 2^{x-1} = 0$

3.9. $e^{-x} = 3 - 2e^x$

3.10. $\log(1-x) = 1$

3.11. $\ln|x| = \ln 3$

3.12. $2\log x = \log 2$

3.13. $x - 2 = \log_{\frac{1}{2}} 2$

3.14. $3e^{x+1} = 2e^{-x}$

Soluções

1.

1.1. $\log_2 32 = \log_2 2^5 = 5$

1.2. $\log_{\frac{1}{3}} 3 = \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{-1} = -1$

1.3. $\log_{\sqrt{5}} 5 = \log_{\sqrt{5}}\left(\sqrt{5}\right)^2 = 2$

1.4. $\log_{\pi}\left(\frac{1}{\pi^2}\right) = \log_{\pi} \pi^{-2} = -2$

1.5. $\log_4 2 = \log_4 4^{\frac{1}{2}} = \frac{1}{2}$

2.

2.1. $5^{3 \times \log_5 2} = (5^{\log_5 2})^3 = 2^3 = 8$

2.2. $\pi^{\log_{32} 1} = \pi^0 = 1$

2.3. $\log_e(\log_{10} 10) = \log_e 1 = 0$

2.4. $\sqrt{2}^{4+\log_2 5} = \sqrt{2}^4 \times \sqrt{2}^{\log_2 5} = \left(2^{\frac{1}{2}}\right)^4 \times \left(2^{\frac{1}{2}}\right)^{\log_2 5} = 2^2 \times \left(2^{\log_2 5}\right)^{\frac{1}{2}} = 4 \times 5^{\frac{1}{2}} = 4\sqrt{5}$

2.5. $\log_{\frac{1}{3}}(3 \times \sqrt{27}) = \log_{\frac{1}{3}}(3 \times \sqrt{27}) = \log_{\frac{1}{3}}\left(3 \times (3^3)^{\frac{1}{2}}\right) = \log_{\frac{1}{3}} 3^{\frac{5}{2}} = \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{-\frac{5}{2}} = -\frac{5}{2}$

2.6. $\log_{125}\sqrt{5} = \log_{125} 5^{\frac{1}{2}} = \log_{125} 125^{\frac{1}{6}} = \frac{1}{6}$

3.

3.1.

$\log_2 x = -2 \Leftrightarrow x = 2^{-2} \wedge x > 0 \Leftrightarrow x = \frac{1}{4}$ C.S. $= \left\{\frac{1}{4}\right\}$

3.2.

$2 = \log_x 3 \Leftrightarrow 3 = x^2 \underset{(x>0)}{\Leftrightarrow} x = \sqrt{3}$ C.S. $= \{\sqrt{3}\}$

3.3.

$\log_{x-1} 4 = 1 \Leftrightarrow x - 1 = 4 \Leftrightarrow x = 5$ C.S. $= \{5\}$

3.4.

$\log_x(2x - 1) = 2 \Leftrightarrow 2x - 1 = x^2 \Leftrightarrow x^2 - 2x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{2 \pm \sqrt{2^2 - 4 \times 1 \times 1}}{2} \Leftrightarrow x = 1$ mas, em $\log_a b$, $a \in \mathbb{R}^+ \setminus \{1\}$,

logo, a equação não tem solução. C.S. $= \varnothing$

3.5.

$x^3 = -4 \Leftrightarrow x = \sqrt[3]{-4}$ C.S. $= \left\{\sqrt[3]{-4}\right\}$

3.6.

$3^x = 4 \Leftrightarrow x = \log_3 4$ C.S. $= \{\log_3 4\}$

3.7.

$e^{2x} = 2 \Leftrightarrow 2x = \ln 2 \Leftrightarrow x = \frac{\ln 2}{2}$ C.S. $= \left\{\frac{\ln 2}{2}\right\}$

3.8.

$$e^{x-1} \cdot 2^x - 2^{x-1} = 0 \Leftrightarrow 2^x(e^{x-1} - 2^{-1}) = 0 \Leftrightarrow \underset{\text{(impossível em } \mathbb{R})}{2^x = 0} \vee e^{x-1} = \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow x - 1 = \ln\left(\frac{1}{2}\right) \Leftrightarrow x = \ln\left(\frac{1}{2}\right) + 1 \Leftrightarrow x = 1 - \ln 2$$

C.S. $= \{1 - \ln 2\}$

3.9.

$$e^{-x} = 3 - 2 \cdot e^x \Leftrightarrow 1 = 3e^x - 2e^{2x} \Leftrightarrow 2e^{2x} - 3e^x + 1 = 0$$

Fazendo $y = e^x$,

$$2y^2 - 3y + 1 = 0 \Leftrightarrow y = \frac{3 \pm \sqrt{9 - 4 \times 2 \times 1}}{2 \times 2} \Leftrightarrow y = \frac{1}{2} \vee y = 1$$

Como $y = e^x$

$$e^x = \frac{1}{2} \vee e^x = 1 \Leftrightarrow x = \ln\left(\frac{1}{2}\right) \vee x = \ln 1 \Leftrightarrow x = \ln\left(\frac{1}{2}\right) \vee x = 0 \Leftrightarrow$$

$$\Leftrightarrow x = -\ln 2 \vee x = 0$$

C.S. $= \{0, -\ln 2\}$

3.10.

$$D = \{x \in \mathbb{R}: 1 - x > 0\} = \left]-\infty, 1\right[$$

$$\log(1 - x) = 1 \Leftrightarrow 1 - x = 10 \Leftrightarrow x = -9$$

C.S. $= \{-9\}$

3.11.

$$D = \{x \Leftrightarrow \mathbb{R}: |x| > 0\} = \mathbb{R}\backslash\{0\}$$

$$\ln|x| = \ln 3 \Leftrightarrow |x| = 3 \Leftrightarrow x = -3 \vee x = 3$$

C.S. $= \{-3, 3\}$

3.12. K

$$D = \mathbb{R}^+$$

$$2 \cdot \log x = \log 2 \Leftrightarrow \log x = \frac{\log 2}{2} \Leftrightarrow \log x = \log\sqrt{2} \Leftrightarrow x = \sqrt{2}$$

C.S. $= \{\sqrt{2}\}$

3.13.

$$D = \mathbb{R}$$

$$x - 2 = \log_{\frac{1}{2}} 2 \Leftrightarrow x = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-1} + 2 \Leftrightarrow x = -1 + 2 \Leftrightarrow x = 1$$

C.S. $= \{1\}$

3.14.

$$D = \mathbb{R}$$

$$3 \times e^{x+1} = 2 \times e^{-x} \Leftrightarrow 3e^{x+1} \cdot e^x = 2 \Leftrightarrow e^{2x+1} = \frac{2}{3} \Leftrightarrow$$

$$\Leftrightarrow 2x + 1 = \ln\left(\frac{2}{3}\right) \Leftrightarrow x = \frac{\ln\left(\frac{2}{3}\right) - 1}{2} \Leftrightarrow x = \frac{1}{2}(\ln 2 - \ln 3 - 1)$$

$$\text{C.S.} = \left\{\frac{1}{2}(\ln 2 - \ln 3 - 1)\right\}$$