Nome do aluno | Nº | Data / / 20

Propriedades da função exponencial

1. Determine:

1.1. $\lim_{x\to+\infty} \frac{-2}{3^x}$

1.2. $\lim_{x\to-\infty} \frac{-2}{3^x}$

1.3. $\lim_{x\to+\infty} 5^{-x}$

2. Sabendo que $3^x = 4$, determine:

2.1. 3^{-x}

2.2. $3^{\frac{x}{2}}$

2.3. 3^{3x}

2.4. 3^{x-1}

3. Resolva, em $\mathbb{R}$, as seguintes expressões:

3.1. $125^{x-1} = 15625$

3.2. $100 \times 3^{0,01x} = 300$

3.3. $7^{-x^2-2x} = 49^{3x+5}$

4. Resolva, em $\mathbb{R}$, as seguintes inequações:

4.1. $2^{3x} > 4^x$

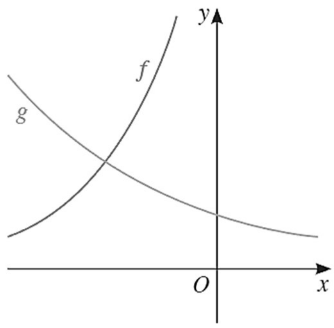
4.2. $5^{4x^2} < 25^{3x}$

4.3. $e^{8x+2} \geq \left(\frac{1}{e}\right)^{3x^2-5}$

5. No referencial cartesiano estão representadas partes dos gráficos das funções f e g definidas analiticamente por:

$$f(x) = 2^{x+3} \qquad g(x) = \left(\sqrt{2}\right)^{-x}$$

Determine, sem recorrer à calculadora, as coordenadas dos pontos de interseção dos gráficos de f e de g.



6. Resolva, em $\mathbb{R}$, as seguintes equações:

6.1. $e^{x+2} = 3e^x$

6.2. $2^{2x+1} + 2^x = 1$

6.3. $e^x + e^{1-x} = 1 + e$

7. Resolva o seguinte sistema de equações:

$$\begin{cases} 2^x + 3^y = 7 \\ 2^{x+1} + 1 = 3^{y+1} \end{cases}$$

8. Resolva, em $\mathbb{R}$, as seguintes inequações:

8.1. $4^x - 6 \times 2^x + 8 < 0$

8.2. $3^{3x-2} \times 9^{2x+1} \leq 27^{x-3}$

8.3. $x^2 2^x - 2^{x+2} < 0$

8.4. $\frac{3e^x - 3}{4^x - 8} \geq 0$

9. Considere a funções f e g, reais de variável real, definidas por:

$$f(x) = 9^x - 6 \qquad g(x) = 3^{x+1} + 2$$

9.1. Determine o contradomínio de f e de g.

9.2. Resolva, por processos analíticos, as condições:

9.2.1. $g(x) = 38 - \frac{81}{3^x}$

9.2.2. $f(x) + g(x) < 0$

9.2.3. $\frac{f(x) - 21}{g(x)} \geq 0$

9.3. Indique:

9.3.1. $\lim_{x \to +\infty} f(x)$

9.3.2. $\lim_{x \to +\infty} g(x)$

10. Determine os seguintes limites:

10.1. $\lim \left(1 + \frac{3}{n}\right)^n$

10.2. $\lim \left(1 - \frac{4}{n}\right)^{2n}$

10.3. $\lim \left(\frac{n+1}{n}\right)^{n+3}$

10.4. $\lim \left(1 + \frac{3}{5n}\right)^{\frac{n}{2}}$

10.5. $\lim \left(\frac{2n+1}{3n+2}\right)^n$

Soluções

1.

1.1.

$$\lim_{x\to+\infty}\frac{-2}{3^x}=-\frac{2}{3^{+\infty}}=-\frac{2}{+\infty}=0$$

1.2.

$$\lim_{x\to+\infty}\frac{-2}{3^x}=-\frac{2}{3^{-\infty}}=-\frac{2}{0^+}=-\infty$$

1.3.

$$\lim_{x\to+\infty}5^{-x}=\lim_{x\to+\infty}\frac{1}{5^x}=\frac{1}{5^{+\infty}}=\frac{1}{+\infty}=0$$

2.

2.1.

$$3^{-x}=(3^x)^{-1}=4^{-1}=\frac{1}{4}$$

2.2.

$$3^{\frac{x}{2}}=(3^x)^{\frac{1}{2}}=4^{\frac{1}{2}}=\sqrt{4}=2$$

2.3.

$$3^{3x}=(3^x)^3=4^3=64$$

2.4.

$$3^{x-1}=3^x\times3^{-1}=4\times\frac{1}{3}=\frac{4}{3}$$

3.

3.1.

$$125^{x-1}=15\,625\Leftrightarrow 125^{x-1}=125^2\Leftrightarrow x-1=2\Leftrightarrow x=3$$

C.S. $=\{3\}$

3.2.

$$100\times3^{0{,}01x}=300\Leftrightarrow 3^{0{,}01x}=3^1\Leftrightarrow 0{,}01x=1\Leftrightarrow x=\frac{1}{0{,}01}\Leftrightarrow x=100$$

C.S. $=\{100\}$

3.3.

$$7^{-x^2-2x}=49^{3x+5}\Leftrightarrow 7^{-x^2-2x}=7^{2(3x+5)}\Leftrightarrow 7^{-x^2-2x}=7^{6x+10}\Leftrightarrow$$

$$\Leftrightarrow -x^2-2x=6x+10\Leftrightarrow -x^2-8x-10=0\Leftrightarrow x^2+8x+10=0\Leftrightarrow$$

$$\Leftrightarrow x=\frac{-8\pm\sqrt{8^2-4\times1\times10}}{2\times1}\Leftrightarrow$$

$$x=\frac{-8\pm\sqrt{64-40}}{2}\Leftrightarrow x=\frac{-8\pm\sqrt{24}}{2}\Leftrightarrow x=\frac{-8\pm2\sqrt{6}}{2}\Leftrightarrow$$

$$\Leftrightarrow x=-4\pm\sqrt{6}$$

C.S. $=\{-4-\sqrt{6},-4+\sqrt{6}\}$

4.

4.1.

$$2^{3x}>4^x\Leftrightarrow 2^{3x}>(2^2)^x\Leftrightarrow 2^{3x}>2^{2x}\Leftrightarrow 3x>2x\Leftrightarrow x>0$$

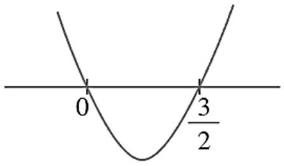
C.S. $=\mathbb{R}^+$

4.2.

$$5^{4x^2} < 25^{3x} \Leftrightarrow 5^{4x^2} < 5^{2 \times 3x} \Leftrightarrow 5^{4x^2} < 5^{6x} \Leftrightarrow 4x^2 < 6x \Leftrightarrow 4x^2 - 6x < 0 \Leftrightarrow$$

$$\Leftrightarrow x > 0 \land x < \frac{3}{2}$$

C.A.: $x(4x - 6) = 0 \Leftrightarrow x = 0 \lor x = \frac{3}{2}$



C.S. $= \left]0, \frac{3}{2}\right[$

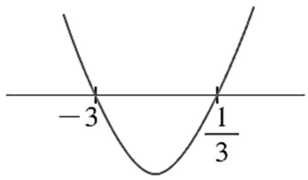
4.3.

$$e^{8x+2} \geqslant \left(\frac{1}{e}\right)^{3x^2-5} \Leftrightarrow e^{8x+2} \geqslant e^{-3x^2+5} \Leftrightarrow 8x + 2 \geqslant -3x^2 + 5 \Leftrightarrow$$

$$\Leftrightarrow 3x^2 + 8x - 3 \geqslant 0 \Leftrightarrow x \leqslant -3 \lor x \geqslant \frac{1}{3}$$

C.A.: $3x^2 + 8x - 3 = 0 \Leftrightarrow x = \frac{-8 \pm \sqrt{64 - 4 \times 3 \times (-3)}}{2 \times 3} \Leftrightarrow$

$$\Leftrightarrow x = -3 \lor x = \frac{1}{3}$$



C.S. $= \left]-\infty, -3\right] \cup \left[\frac{1}{3}, +\infty\right[$

5.

$$f(x) = g(x) \Leftrightarrow 2^{x+3} = \left(\sqrt{2}\right)^{-x} \Leftrightarrow 2^{x+3} = 2^{-\frac{1}{2}x} \Leftrightarrow x + 3 = -\frac{1}{2}x \Leftrightarrow$$

$$\Leftrightarrow x + \frac{x}{2} = -3 \Leftrightarrow \frac{3}{2}x = -3 \Leftrightarrow x = -2$$

$$f(-2) = 2^{-2+3} = 2$$

As coordenadas do ponto de interseção das funções f e g são $I(-2, 2)$.

6.

6.1.

$$e^{x+2} = 3e^x \Leftrightarrow e^x \times e^2 - 3e^x = 0 \Leftrightarrow e^x(e^2 - 3) = 0 \Leftrightarrow \underbrace{e^x = 0}_{\text{(impossível em } \mathbb{R})}$$

C.S. $= \varnothing$

6.2.

$$2^{2x+1} + 2^x = 1 \Leftrightarrow 2^{2x} \times 2 + 2^x = 1 \Leftrightarrow (2^x)^2 \times 2 + 2^x - 1 = 0$$

Fazendo $y = 2^x$,

$$2y^2 + y - 1 = 0 \Leftrightarrow y = \frac{-1 \pm \sqrt{1 - 4 \times 2 \times (-1)}}{2 \times 2} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{1}{2} \lor y = -1$$

como $y = 2^x$

$$2^x = \frac{1}{2} \lor 2^x = -1 \Leftrightarrow 2^x = 2^{-1} \lor 2^x = -1 \Leftrightarrow x = -1$$

C.S. $= \{-1\}$

6.3.

$$e^x + e^{1-x} = 1 + e \Leftrightarrow e^x + \frac{e}{e^x} = 1 + e \Leftrightarrow (e^x)^2 + \mathrm{e} = e^x(1 + e)$$

Fazendo $y = e^x$,

$$y^2 + e - y(1 + e) = 0 \Leftrightarrow y = \frac{1 + e \pm \sqrt{(1 + e)^2 - 4 \times 1 \times e}}{2} \Leftrightarrow$$

$$\Leftrightarrow \frac{1 + e \pm \sqrt{e^2 + 2e + 1 - 4e}}{2} \Leftrightarrow y = \frac{1 + e \pm \sqrt{(e - 1)^2}}{2} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{1 + e \pm (e - 1)}{2} \Leftrightarrow y = \frac{1 + e + e - 1}{2} \lor$$

$$\lor y = \frac{1 + e - e + 1}{2} \Leftrightarrow y = e \lor y = 1$$

Como $y = e^x$:

$e^x = e \quad \lor \quad e^x = 1 \Leftrightarrow x = 1 \lor x = 0$

C.S. $= \{0, 1\}$

7.

$$\begin{cases} 2^x + 3^y = 7 \\ 2^{x+1} + 1 = 3^{y+1} \end{cases} \Leftrightarrow \begin{cases} 3^y = 7 - 2^x \\ 2^{x+1} + 1 = 3^y \times 3 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ 2^{x+1} + 1 = (7 - 2^x) \times 3 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \text{———} \\ 2^{x+1} + 1 = 21 - 3 \times 2^x \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ 2^x \times 2 + 3 \times 2^x = 20 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \text{———} \\ 5 \times 2^x = 20 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ 2^x = 4 \end{cases} \Leftrightarrow \begin{cases} \text{———} \\ 2^x = 2^2 \end{cases} \Leftrightarrow \begin{cases} 3^y = 7 - 2^2 \\ x = 2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 3^y = 3 \\ \text{———} \end{cases} \Leftrightarrow \begin{cases} y = 1 \\ x = 2 \end{cases}$$

C.S. $= \{(2, 1)\}$

8.

8.1.

$4^x - 6 \times 2^x + 8 < 0 \Leftrightarrow (2^x)^2 - 6 \times 2^x + 8 < 0$

Fazendo $y = 2^x$, tem-se $y^2 - 6y + 8 < 0 \Leftrightarrow y > 2 \land y < 4$.

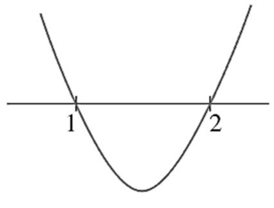
Como $y = 2^x$:

$2^x > 2 \land 2^x < 4 \Leftrightarrow x > 1 \land x < 2$

C.A.:

$$y = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 8}}{2} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{6 \pm \sqrt{4}}{2} \Leftrightarrow y = 2 \lor y = 4$$



C.S. $=]1, 2[$

8.2. L

$$3^{3x-2} \times 9^{2x+1} \leqslant 27^{x-3} \Leftrightarrow 3^{3x-2} \times 3^{2(2x+1)} \leqslant 3^{3(x-3)} \Leftrightarrow$$

$$\Leftrightarrow 3^{7x} \leqslant 3^{3x-9} \Leftrightarrow 7x \leqslant 3x - 9 \Leftrightarrow$$

$$\Leftrightarrow 4x \leqslant -9 \Leftrightarrow x \leqslant -\frac{9}{4}$$

$$\text{C.S.} = \left]-\infty, -\frac{9}{4}\right]$$

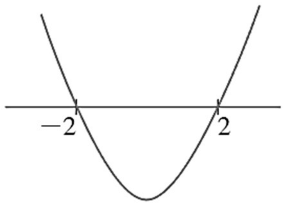
8.3.

$x^2 \times 2^x - 2^{x+2} < 0 \Leftrightarrow x^2 \times 2^x - 2^x \times 2^2 < 0 \Leftrightarrow$

$\Leftrightarrow (x^2 - 4) \times 2^x < 0 \underset{(2^x > 0)}{\Leftrightarrow} x^2 - 4 < 0 \Leftrightarrow$

$\Leftrightarrow x > -2 \wedge x < 2$

C.S. $=]-2, 2[$



8.4.

$$\frac{3e^x - 3}{4^x - 8} \geqslant 0$$

C.A.: $3e^x - 3 = 0 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0$

$4^x - 8 = 0 \Leftrightarrow 2^{2x} = 2^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2}$

	$-\infty$	0		$\frac{3}{2}$	$+\infty$
$3e^x - 3$	$-$	0	$+$	$+$	$+$
$4^x - 8$	$-$	$-$	$-$	0	$+$
$\frac{3e^x - 3}{4^x - 8}$	$+$	0	$-$	n.d.	$+$

C.S. $= \left]-\infty, 0\right] \cup \left]-\frac{3}{2}; +\infty\right[$

9.

9.1.

$D'_f =]-6; +\infty[$ $\qquad$ $D'_g =]2, +\infty[$

9.2.

9.2.1.

$g(x) = 38 - \frac{81}{3^x} \Leftrightarrow 3^{x+1} + 2 = 38 - \frac{81}{3^x} \Leftrightarrow$

$\Leftrightarrow 3^{x+1} + 2 = 38 - 81 \times 3^{-x} \Leftrightarrow 3^{x+1} + 81 \times 3^{-x} - 36 = 0 \Leftrightarrow$

$\Leftrightarrow 3^{x \times 3} + \frac{81}{3^x} - 36 = 0 \Leftrightarrow 3^{2x} \times 3 + 81 - 36 \times 3^x = 0$

Fazendo $y = 3^x$,

$3y^2 - 36y + 81 = 0 \Leftrightarrow y^2 - 12y + 27 = 0 \Leftrightarrow$

$\Leftrightarrow y = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 1 \times 27}}{2} \Leftrightarrow y = 3 \vee y = 9$

Como $y = 3^x$,

$3^x = 3 \vee 3^x = 9 \Leftrightarrow x = 1 \vee x = 2$

C.S. $= \{1, 2\}$

9.2.2. L

$f(x) + g(x) < 0 \Leftrightarrow 9^x - 6 + 3^{x+1} + 2 < 0 \Leftrightarrow$

$\Leftrightarrow 3^{2x} - 6 + 3^x \times 3 + 2 < 0$

Fazendo $y = 3^x$:

$y^2 + 3y - 4 < 0 \Leftrightarrow y = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times (-4)}}{2} \Leftrightarrow$

$\Leftrightarrow y > -4 \wedge y < 1$

como $y = 3^x$,

$3^x > -4 \wedge 3^x < 1 \Leftrightarrow 3^x < 3^0 \Leftrightarrow x < 0$

(condição universal)

C.S. $=]-\infty, 0[= \mathbb{R}^-$

9.2.3. L

$$\frac{f(x) - 21}{g(x)} \geqslant 0 \Leftrightarrow \frac{9^x - 6 - 21}{3^{x+1} + 2} \geqslant 0 \Leftrightarrow \frac{9^x - 27}{3^{x+1} + 2} \geqslant 0$$

como $3^{x+1} + 2 > 0$, o sinal de $9^x - 27$ determina o sinal de $\frac{9^x - 27}{3^{x+1} + 2}$

$$\frac{9^x - 27}{3^{x+1} + 2} \geqslant 0 \Leftrightarrow 9^x - 27 \geqslant 0 \Leftrightarrow 3^{2x} \geqslant 3^3 \Leftrightarrow 2^x \geqslant 3 \Leftrightarrow x \geqslant \frac{3}{2}$$

C.S. $= \left[\frac{3}{2}, +\infty\right[$

9.3.

9.3.1.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (9^x - 6) = 9^{-\infty} - 6 = 0 - 6 = -6$$

9.3.2.

$$\lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} (3^{x+1} + 2) = 3^{+\infty} + 2 = +\infty$$

10.

10.1.

$$\lim\left(1 + \frac{3}{n}\right)^n = e^3$$

10.2.

$$\lim\left(1 - \frac{4}{n}\right)^{2n} = \lim\left[\left(1 - \frac{4}{n}\right)^n\right]^2 = (e^{-4})^2 = \mathrm{e}^{-8}$$

10.3.

$$\lim\left(\frac{n+1}{n}\right)^{n+3} = \lim\left(1 + \frac{1}{n}\right)^{n+3} = \lim\left[\left(1 + \frac{1}{n}\right)^n \times \left(1 + \frac{1}{n}\right)^3\right] = e \times 1 = e$$

10.4.

$$\lim\left(1 + \frac{3}{5n}\right)^{\frac{n}{2}} = \lim\left[\left(1 + \frac{\frac{3}{5}}{n}\right)^n\right]^{\frac{1}{2}} = \left[e^{\frac{3}{5}}\right]^{\frac{1}{2}} = e^{\frac{3}{10}}$$

10.5.

$$\lim\left(\frac{2n+1}{2n+2}\right) = \lim\left(\frac{2n}{3n}\right) = \frac{2}{3}, \quad 0 < \frac{2}{3} < 1$$

Logo: $\lim\left(\frac{2n+1}{3n+2}\right)^n = 0$